Eur. Phys. J. C 21, 313–318 (2001)
Digital Object Identifier (DOI) 10.1007/s100520100724

THE EUROPEAN PHYSICAL JOURNAL C

Matrix elements of four-quark operators relevant to life time difference $\Delta\Gamma_{B_s}$ from QCD sum rules

C.-S. Huang[1], Ailin Zhang[1], S.-L. Zhu[1,2]

[1] Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, P.R. China
[2] Department of Physics, University of Connecticut, Storrs, CT 06269, USA

Received: 23 April 2001 / Revised version: 27 May 2001 /
Published online: 19 July 2001 –

Abstract. We extract the matrix elements of four-quark operators $O_{L,S}$ relevant to the B_s and $\bar{B}_s$ life time difference from QCD sum rules. We find that the vacuum saturation approximation works reasonably well, i.e., within 10%. We discuss the implications of our results and compare them with a recent lattice QCD determination.

1 Introduction

Recently, results on CP violation in B_d–$\bar{B}_d$ mixing have been reported by the BaBar and Belle Collaborations [1] at the ICHEP2000 Conference. More experiments on B physics have been planned at present and future B factories [2]. Theoretical efforts to improve predictions and reduce uncertainties are expected and needed. It is well known that mixing in neutral B meson systems provides a good place to examine CP violation as well as flavor physics in the standard model and beyond. For example, the mass difference between the mass eigenstates of the neutral B_d meson, ΔM_{B_d}, gives an important constraint on the CKM matrix element V_{td} and gives a first indication of the large mass of the top quark. Similarly, the mass difference between the mass eigenstates of the neutral B_s meson, ΔM_{B_s}, which will be precisely measured in the near future would give a valuable constraint on the CKM matrix element V_{ts}. Another important observable for mixing in neutral B meson systems is the life time difference between the mass eigenstates of the neutral B mesons, $\Delta\Gamma_{B_d}$ or $\Delta\Gamma_{B_s}$. The ratio $|V_{ts}/V_{td}|^2$ can be extracted from the measurement of $\Delta\Gamma_{B_s}$ [3]. The width difference of the B_d mesons is CKM suppressed and consequently not easy to be observed. In contrast, for B_s mesons the width difference is large enough to be measured [4] and has recently been measured [5] with low statistics. Hopefully, it will be measured with high statistics in the near future.

As usual, the light B_s^{L} and heavy B_s^{H} mass eigenstates are defined by

$$|B_s^{\mathrm{L,H}}\rangle = p|B_s^0\rangle \pm q|\bar{B}_s^0\rangle,$$

where $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$ are the flavor eigenstates. The mass difference and the width difference between the physical states are given by

$$\Delta m \equiv M_{\mathrm{H}} - M_{\mathrm{L}}, \quad \Delta\Gamma \equiv \Gamma_{\mathrm{H}} - \Gamma_{\mathrm{L}}.$$

Because $|\Gamma_{12}| \ll |M_{12}|$ for B_s mesons [6], to leading order in $|\Gamma_{12}/M_{12}|$, $\Delta m_B = 2|M_{12}|$, $\Delta\Gamma_B = 2\Re(M_{12}\Gamma_{12}^*)/|M_{12}|$. Neglecting very small CP violating corrections, the width difference for B_s mesons in SM has been given [6,7]:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 \times [0.006B(m_b) + 0.150B_S(m_b) - 0.063], \quad (1)$$

where f_{B_s} is the decay constant of B_s, and B and B_S are the bag parameters related to the four-quark operators O_L and O_S (see below). These hadronic quantities need to be calculated by non-perturbative methods such as lattice methods, QCD sum rules, the Bethe–Salpeter approach, etc.

Similar quantities related to B_d^0–$\bar{B}_d^0$ mixing have been estimated by Narison et al. within the traditional QCD sum rules approach [8] at order α_s. Their conclusion is that the vacuum saturation values $B_B \simeq B_{B^*} \simeq 1$ are satisfied within 15%. Their sum rules are constructed through two-point correlation functions and depend on some phenomenological assumptions. In this letter we shall calculate the matrix elements of four-quark operators relevant to the B_s meson life time difference through QCD sum rules in HQET. The sum rules are constructed with three-point correlation functions. Our calculation is carried out at the leading order in the $1/m_b$ expansion in HQET for simplicity. In [8] the effects of condensates are absorbed like other factorizable corrections into the contribution to f_B and the available result of [9] (though not explicitly stated) has been used as the effects are small compared to the perturbative corrections. In our sum rules the non-perturbative contributions of the condensates are explicitly included and the numerical results confirm the smallness of these corrections (see below).

2 Theoretical formalism

We employ the following three-point Green's function:

$$\begin{aligned}&\Gamma^{O}(\omega,\omega')\\&=\mathrm{i}^2\int \mathrm{d}x\mathrm{d}y\mathrm{e}^{\mathrm{i}k'\cdot x+\mathrm{i}k\cdot y}\\&\quad\times\langle 0|\mathcal{T}[\bar{s}(x)\gamma_5 h_v^{(b)}(x)]O_{L,S}(0)[\bar{s}(y)\gamma_5 h_v^{(b)}(y)]|0\rangle,\end{aligned}\quad (2)$$

where $\omega=v\cdot k$, $\omega'=v\cdot k'$; $h_v^{(b)}$ is the b-quark field in the HQET with velocity v. $O_{L,S}$ denote the color-singlet four-quark operators. They are

$$O_L=\bar{b}\gamma^{\mu}(1-\gamma_5)s\bar{b}\gamma_{\mu}(1-\gamma_5)s, \quad (3)$$

$$O_S=\bar{b}(1-\gamma_5)s\bar{b}(1-\gamma_5)s. \quad (4)$$

In terms of the hadronic expression, the correlator in (2) reads

$$\Gamma^{O}(\omega,\omega')=\frac{F_{B_s}^2}{4}\frac{\langle\bar{B}_s|Q_{L,S}|B_s\rangle}{(\bar{\Lambda}-\omega)(\bar{\Lambda}-\omega')}+\text{resonances}, \quad (5)$$

where $\bar{\Lambda}=m_B-m_b$ and F_{B_s} is the B_s decay constant in the leading order of the heavy quark expansion defined as

$$\langle 0|\bar{s}(0)\gamma_5 h_v^{(b)}(0)|B_s\rangle=-\mathrm{i}\sqrt{m_Q}F_{B_s}. \quad (6)$$

Note that f_{B_s} in (1) is defined by

$$\langle 0|\bar{s}\gamma^{\mu}\gamma_5 b|B_s^0\rangle=-\mathrm{i}f_{B_s}p^{\mu}. \quad (7)$$

In order to eliminate the contribution from the non-diagonal single pole terms and suppress the continuum contribution in (5), we perform a double Borel transformation on the correlator. This transformation is defined as

$$\begin{aligned}\hat{B}=&\lim_{\substack{-\omega\to\infty\\ n\to\infty\\ \tilde{\tau}\equiv\frac{-\omega}{n}\ \text{fixed}}}\ \lim_{\substack{-\omega'\to\infty\\ m\to\infty\\ \tilde{\tau}'\equiv\frac{-\omega'}{m}\ \text{fixed}}}\frac{(-\omega)^{n+1}}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}\omega}\right)^{n}\\&\times\frac{(-\omega')^{m+1}}{m!}\left(\frac{\mathrm{d}}{\mathrm{d}\omega'}\right)^{m}.\end{aligned}\quad (8)$$

There are two Borel parameters, $\tilde{\tau}$ and $\tilde{\tau}'$, which appear symmetrically, so $\tilde{\tau}=\tilde{\tau}'=2T$ are taken in the following analysis.

On the other hand, the correlator can be calculated at the quark gluon level. For example, for O_L we may rewrite the right hand side of (2) as

$$\begin{aligned}-2\int &\mathrm{d}x\mathrm{d}y\mathrm{e}^{\mathrm{i}k'\cdot x+\mathrm{i}k\cdot y}\Big\{-\mathrm{Tr}[\gamma_5\cdot\mathrm{i}S_b^{mi}(x)\cdot\gamma^{\mu}(1-\gamma_5)\\&\cdot\mathrm{i}S_s^{\mathrm{in}}(-y)\cdot\gamma_5\mathrm{i}S_b^{nj}(y)\cdot\gamma_{\mu}(1-\gamma_5)\cdot\mathrm{i}S_s^{jm}(-x)]\\&+\mathrm{Tr}[\mathrm{i}S_s^{im}(-x)]\cdot\gamma_5\cdot\mathrm{i}S_b^{mi}(x)\cdot\gamma^{\mu}(1-\gamma_5)]\\&\cdot\mathrm{Tr}[\mathrm{i}S_s^{jn}(-y)]\cdot\gamma_5\cdot\mathrm{i}S_b^{nj}(y)\cdot\gamma_{\mu}(1-\gamma_5)]\Big\},\end{aligned}\quad (9)$$

where $\mathrm{i}S_s^{jn}(x)$ is the full strange quark propagator with both perturbative term and condensates; i,j etc. are color indices. $\mathrm{i}S_b^{nj}(x)$ is the leading order heavy quark propagator, which has a very simple form in coordinate space:

$$\mathrm{i}S_b^{ij}(x)=\delta^{ij}\int_0^{\infty}\mathrm{d}t\delta(x-vt). \quad (10)$$

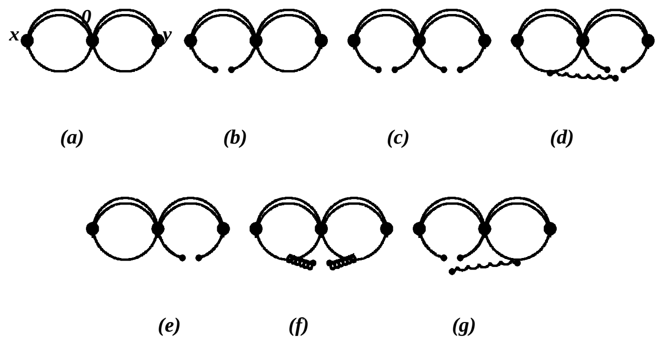

Fig. 1a–g. Dominant non-vanishing Feynman diagrams for $\Gamma^{O}(\omega,\omega')$

Note that the structure of the color flow is quite different for the two terms in (9). For the perturbative part the first and second term is proportional to N_c and N_c^2, respectively, where $N_c=3$ is the QCD color number. In the limit of $N_c\to\infty$, the second term dominates! As shown below, the non-factorizable contribution in Figs. 1d,f,g has a color structure different from the factorizable terms in Figs. 1a,b,c,e. The condensates up to dimension six are kept in our calculation. We also expand the strange quark propagator and keep the perturbative term of order $\mathcal{O}(m_s)$. The calculation is standard and we simply present the final results after performing the double Borel transformation.

3 Duality assumption

We may write the dispersion relation for the three-point correlator $\Gamma(\omega,\omega')$ as

$$\Pi(\omega,\omega')=\frac{1}{\pi^2}\int_0^{\infty}\mathrm{d}\nu\int_0^{\infty}\mathrm{d}\nu'\frac{\mathrm{Im}\Pi(\nu,\nu')}{(\nu-\omega)(\nu'-\omega')}. \quad (11)$$

In order to subtract the continuum contribution, we have to invoke the quark–hadron duality assumption and approximate the continuum by the integral over the perturbative spectral density above a certain energy threshold ω_c.

With the redefinition of the integral variables

$$\begin{aligned}\nu_+&=\frac{\nu+\nu'}{2},\\ \nu_-&=\frac{\nu-\nu'}{2},\end{aligned}\quad (12)$$

the integration becomes

$$\int_0^{\infty}\mathrm{d}\nu\int_0^{\infty}\mathrm{d}\nu'\cdots=2\int_0^{\infty}\mathrm{d}\nu_+\int_{-\nu_+}^{\nu_+}\mathrm{d}\nu_-\cdots \quad (13)$$

It is in ν_+ that the quark–hadron duality is assumed [10–12]:

$$\text{higher states} = \frac{2}{\pi^2}\int_{\omega_c}^{\infty}\mathrm{d}\nu_+\int_{-\nu_+}^{\nu_+}\mathrm{d}\nu_-\frac{\mathrm{Im}\Pi(\nu,\nu')}{(\nu-\omega)(\nu'-\omega')}. \tag{14}$$

This kind of assumption was suggested in calculating the Isgur–Wise function in [11] and was argued for in [12]. As pointed out in [10,12], in calculating three-point functions the duality is valid after integrating the spectral density over the "off-diagonal" variable $\nu_- = (1/2)(\nu-\nu')$. Such a duality assumption is favored over the naive one:

$$\text{higher states} = \frac{1}{\pi^2}\int_{\omega_c}^{\infty}\mathrm{d}\nu\int_{\omega_c}^{\infty}\mathrm{d}\nu'\frac{\mathrm{Im}\Pi(\nu,\nu')}{(\nu-\omega)(\nu'-\omega')}. \tag{15}$$

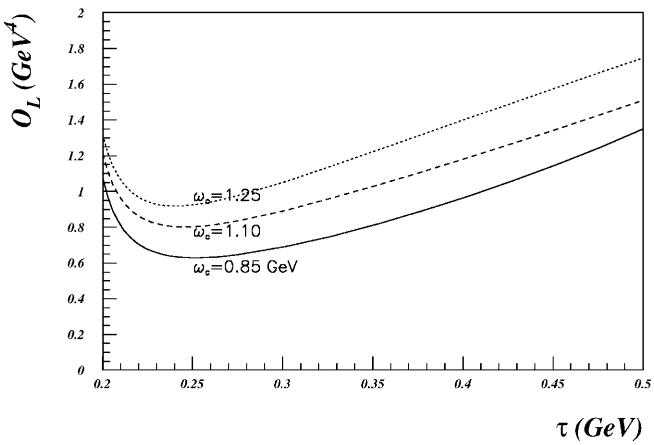

Fig. 2. The dependence of $\langle \bar{B}_s|O_L|B_s\rangle$ on T, ω_c

4 QCD sum rules

The spectral density $\rho_{L,S}(s_1,s_2)$ of the perturbative term reads

$$\rho_L(s_1,s_2) = \frac{N_c(N_c+1)}{2\pi^4}s_1s_2[s_1s_2+m_s(s_1+s_2)], \tag{16}$$

$$\rho_S(s_1,s_2) = \frac{N_c(2N_c-1)}{4\pi^4}s_1s_2[s_1s_2+m_s(s_1+s_2)]. \tag{17}$$

The sum rule for $\langle \bar{B}_s|O_{L,S}|B_s\rangle$ after the inclusion of the condensates and the integration with the variable ν_- is

$$\begin{aligned}
&\frac{F_{B_s}^2}{4}\langle \bar{B}_s|O_L|B_s\rangle \exp\left(-\frac{\bar{\Lambda}}{T}\right)\\
&= \frac{N_c(N_c+1)}{\pi^4}\left\{\int_0^{\omega_c}\mathrm{d}\nu \exp\left(-\frac{\nu}{T}\right)\left[\frac{16}{15}\nu^5+\frac{8}{3}m_s\nu^4\right]\right.\\
&\quad\left.+\frac{4}{3}a_sT^3\left(1-\frac{m_0^2}{64T^2}\right)+\frac{1}{6}m_sa_sT^2+\frac{a_s^2}{288}\right\}\\
&\quad-\frac{N_c^2-1}{256\pi^4}[2T^2\langle g_s^2G^2\rangle + a_sm_0^2T],
\end{aligned} \tag{18}$$

where $a_s = -(2\pi)^2\langle \bar{s}s\rangle$ and we have used the factorization assumption for the four-quark condensates. Similarly, we have

$$\begin{aligned}
&\frac{F_{B_s}^2}{4}\langle \bar{B}_s|O_S|B_s\rangle \exp\left(-\frac{\bar{\Lambda}}{T}\right)\\
&= \frac{N_c(2N_c-1)}{2\pi^4}\left\{\int_0^{\omega_c}\mathrm{d}\nu \exp\left(-\frac{\nu}{T}\right)\left[\frac{16}{15}\nu^5+\frac{8}{3}m_s\nu^4\right]\right.\\
&\quad\left.+\frac{4}{3}a_sT^3\left(1-\frac{m_0^2}{64T^2}\right)+\frac{1}{6}m_sa_sT^2+\frac{a_s^2}{288}\right\}\\
&\quad-\frac{N_c^2-1}{512\pi^4}[2T^2\langle g_s^2G^2\rangle + a_sm_0^2T].
\end{aligned} \tag{19}$$

We want to emphasize that in (18) and (19) the terms with the color factor $N_c(N_c+1)$ and $N_c(2N_c-1)$ come from the factorizable diagrams in Figs. 1a,b,c,e. The non-factorizable contribution has a color factor $(N_c^2-1)/2$ which comes from the summation over the color factor, $\mathrm{Tr}\,[(\lambda^a/2)(\lambda^a/2)] = (N_c^2-1)/2$, in Figs. 1d,f,g. A second observation is that the factorizable terms are all positive while the non-factorizable pieces are negative.

Now we turn to the numerical analysis. The decay constant and the binding energy of the B_s meson at the leading order of the heavy quark expansion can be obtained from the mass sum rule [13]. We have

$$\begin{aligned}
F_{B_s}^2\exp\left(-2\frac{\bar{\Lambda}}{M}\right) &= \frac{3}{8\pi^2}\int_0^{s_0}\mathrm{d}s s(s+2m_s)\mathrm{e}^{-s/M}\\
&\quad-\langle \bar{s}s\rangle\left(1-\frac{m_0^2}{4M^2}\right).
\end{aligned} \tag{20}$$

Note that $M = 2T, s_0 = 2\omega_c$. We have not included α_s corrections in (20), because they are also neglected in the sum rule for $\langle \bar{B}_s|O_{L,S}|B_s\rangle$, (18) and (19). The values of the parameters are calculated to be $F_{B_s} = (0.49\pm0.1)\,\mathrm{GeV}^{3/2}$, $\bar{\Lambda} = (0.68\pm0.1)\,\mathrm{GeV}$ with the threshold s_0 to be $(2.2\pm0.3)\,\mathrm{GeV}$ and the Borel parameter M in the window (0.65–1.05) GeV [13]. Numerically, we use the following values of the condensates:

$$\begin{aligned}
\langle \bar{s}s\rangle &\simeq -0.8\times(0.23\,\mathrm{GeV})^3,\\
\langle g_s^2G^2\rangle &\simeq 0.48\,\mathrm{GeV}^4,\\
\langle g\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle &\equiv m_0^2\langle \bar{s}s\rangle, m_0^2 \simeq 0.8\,\mathrm{GeV}^2.
\end{aligned} \tag{21}$$

For the strange quark mass we use $m_s = 0.15\,\mathrm{GeV}$.

In order to minimize the dependence of the parameters we divide (18) and (19) by (20) to extract the matrix elements, the variation of which with ω_c and T are given in Figs. 2 and 3. The sum rule window is $T = (0.2–0.5)\,\mathrm{GeV}$, which is almost the same as in the two-point correlator sum rule. We obtain

$$\langle \bar{B}_s|O_L|B_s\rangle = (0.85\pm0.20)\,\mathrm{GeV}^4, \tag{22}$$

$$|\langle \bar{B}_s|O_S|B_s\rangle| = (0.55\pm0.15)\,\mathrm{GeV}^4, \tag{23}$$

where the central value corresponds to $T = 0.3\,\mathrm{GeV}$ and $\omega_c = 1.1\,\mathrm{GeV}$. The uncertainty includes the variation with T and ω_c. The bag parameters B and B_S are defined by

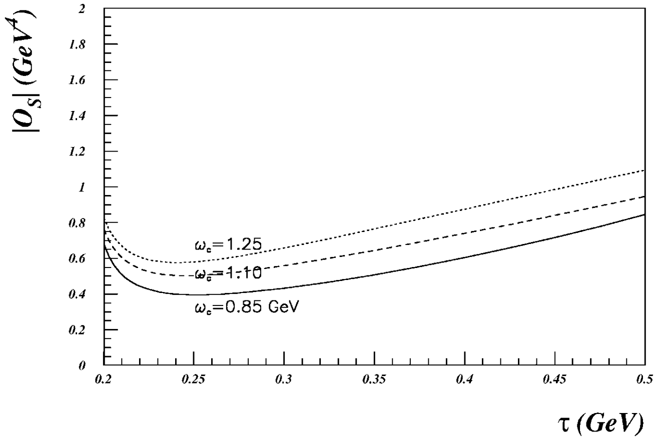

Fig. 3. The variation of $|\langle \bar{B}_s|O_S|B_s\rangle|$ with T, ω_c

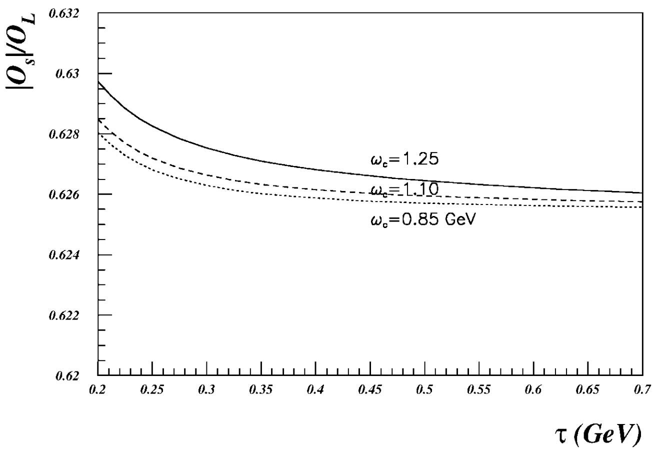

Fig. 4. The variation of $\mathcal{R}$ with T, ω_c

$$\langle \bar{B}_s|O_L|B_s\rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B,$$
$$\langle \bar{B}_s|O_S|B_s\rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2} B_S. \quad (24)$$

After taking into account the scale dependence (and the mixing effect of the renormalization) of these four-quark operators, which is described in [7], we obtain $B = 0.82$, $B_s = 0.78$. To get these numerical results, $f_{B_s} = F_{B_s}/m_b^{1/2} = 0.21\,\mathrm{GeV}$, $M_{B_s} = 5.37\,\mathrm{GeV}$, $\bar{m}_b(m_b) = 4.4\,\mathrm{GeV}$, $\bar{m}_s = 0.2\,\mathrm{GeV}$ are used [7].

The ratio of these two matrix elements is very interesting. We divide (20) by (19) to extract the numerical value of the ratio. In such a way the dependence on the Borel parameter and the continuum threshold is minimized as can be clearly seen in Fig. 4. Within the accuracy of QSR the curve in Fig. 4 is flat. The ratio is practically the same in the working region of T and ω_c. It reads

$$\mathcal{R} = \frac{|\langle \bar{B}_s|O_S|B_s\rangle|}{\langle \bar{B}_s|O_L|B_s\rangle} = (0.63 \pm 0.13). \quad (25)$$

In our numerical calculation, the contribution of the perturbative term is about 45–65% of the total contributions in the preferred Borel variable region. We have used a factorization approximation for the four-quark condensates in the numerical calculations. This may introduce some uncertainty. We may introduce a scale factor κ to indicate the deviation from the factorization approximation as in [14]. In our calculations of the sum rules the $1/M_b$ corrections in HQET have not been included, which may bring about a deviation from the numerical results of the matrix elements. However, for the ratio of the two matrix elements, we expect little change to the above analysis. Our numerical results are not sensitive to the mass of the strange quark. Actually, the effects due to the strange quark are very small, so that the results for B_s are almost the same as those for B_d.

We now make a remark on the usual factorization assumption. In our Feynman diagram (Fig. 1) calculations, the contributions of non-factorizable diagrams are around $-6\%, -7\%$ for $\langle \bar{B}_s|O_S|B_s\rangle$ and $\langle \bar{B}_s|O_L|B_s\rangle$ respectively, which means that the factorization approach works well, even though our calculations are limited to the leading order in the $1/m_b$ expansion in HQET. That is, the conclusion in [8] remains unchanged when the non-perturbative condensate contributions are taken into account. If one considers α_s corrections, there is only one non-factorizable perturbative diagram, in which the gluon line in Fig. 1f is connected, in the fixed-point gauge in the leading $1/M_b$ expansion. However, radiative corrections are generally of high order in α_s/π compared to the leading order, and the fact is that the perturbative term is about 45–65% of the whole contribution, so the contribution from the diagram can be neglected compared to those in Figs. 1d,f,g. The case here is different from that in the calculation of the matrix elements of the four-quark operators, relevant to the life time difference between the heavy mesons, where the flavor changes by $\Delta F = 0$. In that case, the perturbative contribution vanishes [15], and we cannot predict naively how large the radiative correction is compared to the non-perturbative terms.

5 The B_s and $\bar{B}_s$ decay width difference

The complete expression for $\Delta\Gamma_{B_s}$ with short-distance coefficients at NLO in QCD is given by [7]

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{16\pi^2 B(B_s \to Xe\nu)}{g(z)\tilde{\eta}_{\mathrm{QCD}}} \frac{f_{B_s}^2 M_{B_s}}{m_b^3} |V_{cs}|^2 \quad (26)$$
$$\cdot \left(G(z)\frac{8}{3}B + G_S(z)\frac{M_{B_s}^2}{(\bar{m}_b+\bar{m}_s)^2}\frac{5}{3}B_S + \sqrt{1-4z}\,\delta_{1/m} \right),$$

where

$$G(z) = F(z) + P(z) \quad \text{and} \quad G_S(z) = -(F_S(z) + P_S(z)). \quad (27)$$

and F, P, F_S, P_S can be found in [7]. We eliminated the total decay rate Γ_{B_s} in favor of the semileptonic branching ratio $B(B_s \to Xe\nu)$, as in [6]. This cancels the dependence of $(\Delta\Gamma/\Gamma)$ on V_{cb} and introduces the phase space function

$$g(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z, \quad (28)$$

as well as the QCD correction factor [16]

$$\tilde{\eta}_{\rm QCD} = 1 - \frac{2\alpha_{\rm s}(m_b)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-\sqrt{z})^2 + \frac{3}{2}\right]. \quad (29)$$

One can also express the width difference as

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = \left(\tau_{B_s}\Delta m_{B_d}\frac{m_{B_s}}{m_{B_d}}\right)^{(\rm exp.)}\left|\frac{V_{ts}}{V_{td}}\right|^2 K \cdot (G(z) - G_S(z)\mathcal{R}(m_b) + \tilde{\delta}_{1/m})\xi^2, \quad (30)$$

where

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}, \quad (31)$$

K is (7) in [17],

$$\tilde{\delta}_{1/m} = f_{B_s}^2 M_{B_s}^2 \delta_{1/m}, \quad (32)$$

and $\tilde{\delta}_{1/m}$ represents the $1/m_b$ corrections and can be found in [6].

It is clear from the above equation that besides the ratio $\mathcal{R}$ of the matrix elements of the four-quark operators, which are those we have calculated in our paper, we only use the experimental B_d meson mass difference, which is known with a tiny error: [18]

$$(\Delta m_{B_d})^{(\rm exp.)} = 0.484(15)\,{\rm ps}^{-1}, \quad (33)$$

and another ratio of the hadronic matrix elements, ξ, which is rather accurately determined in lattice simulations [19,20].

As is well known, the quantities in (1) are calculated at the scale $O(m_b)$, while our result (25) is calculated at the hadron scale $\mu_{\rm had}$. Therefore, we have to consider the renormalization scale dependence of those four-quark operators. The anomalous dimension matrix of these operators has been given in [7]. Using the anomalous dimension matrix and following the standard way, we obtain the scale dependence of $\mathcal{R}$,

$$\mathcal{R}(m_b) = 1.69\mathcal{R}(\mu_{\rm had}) + 0.03, \quad (34)$$

where $\mathcal{R}(\mu_{\rm had})$ is defined by (25). To obtain the numerical result, $m_b = 4.8\,{\rm GeV}$ and $\mu_{\rm had} = 1.0\,{\rm GeV}$ have been used. It is obvious from (34) that the result heavily depends on the renormalization scale.

Numerically, we have

$$\begin{aligned}\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} &= [(0.5\pm 0.1) + (13.8\pm 2.8)\mathcal{R}(m_b) \\ &\quad +(15.7\pm 2.8)(-0.55\pm 0.17)]\times 10^{-2} \\ &= (7.0\pm 0.8)\times 10^{-2}. \end{aligned} \quad (35)$$

Clearly such a life time difference is compatible with the existing literature. It is interesting to compare our result to the two recent lattice QCD calculations: $\Delta\Gamma_{B_s}/\Gamma_{B_s} = (10.7\pm 2.6\pm 1.4\pm 1.7)\times 10^{-2}$ in [21] and $\Delta\Gamma_{B_s}/\Gamma_{B_s} = (4.7\pm 1.5\pm 1.6)\times 10^{-2}$ in [17]. In (35) the numerical value of $\tilde{\delta}_{1/m}$, which corresponds to the $1/m_b$ correction in the short-distance expansion of the operator product $H_{\rm eff}(x)H_{\rm eff}(0)$ [7], has been taken as -0.55 [17]. If it is taken as -0.30, one has $\Delta\Gamma/\Gamma = 10.9\times 10^{-2}$, larger than 7.0×10^{-2}, while in the case of [17], $\Delta\Gamma/\Gamma$ would remain in the 10% range with the change from -0.55 to -0.30. That is, the sensitivity to the final term in (35), i.e., the $1/m_b$ correction, increases in our result. Without a good control of this correction, a precise determination of the life time difference is impossible.

6 Conclusion and discussion

In summary, we have calculated the matrix elements of the four-quark operators relevant to the B_s meson life time difference with QCD sum rules in HQET. The sum rules are constructed with three-point correlators and both the perturbative and non-perturbative contribution are taken into account. Our result shows that the usual factorization assumption is indeed a good approximation. The numerical results show that the sum rules of those operators have a good platform. The perturbative contribution to sum rules are about 45–65% of the total contribution. Our results are not sensitive to m_s. The life difference $\Delta\Gamma_{B_s}/\Gamma_{B_s}$ is found to be around $(7.0\pm0.8)\times10^{-2}$. This result is compatible with results predicted by lattice calculations. The $\alpha_{\rm s}$ corrections have not been taken into account in the sum rules and they will definitely have effects on the resulting numerical values. To get a more accurate prediction, the $\alpha_{\rm s}$ corrections should be taken into account; this is beyond the content of this letter.

Acknowledgements. The work was partly supported by the National Natural Science Foundation of China.

References

1. Plenary talks presented by D. Hitlin (BaBar Collaboration), H. Aihara (Belle Collaboration) at ICHEP2000, Osaka, Japan, 31 July 2000 (to appear in the Proceedings)
2. N. Ellis (LHC), at IV International Conference on Hyperons, Charm and Beauty Hadrons, Valencia, Spain, 29 June 2000 (to appear in the Proceedings)
3. T.E. Browder, S. Pakvasa, Phys. Rev. D **52**, 3123 (1995)
4. I. Bigi et al., in *B* Decays, second edition, edited by S. Stone (World Scientific, Singapore 1994)
5. For a recent review, see S. Wasserbaech (ALEPH), at IV International Conference on Hyperons, Charm and Beauty Hadrons, Valencia, Spain, 29 June 2000 (to appear in the Proceedings)
6. M. Beneke, G. Buchalla, I. Dunietz, Phys. Rev. D **54**, 4419 (1996)
7. M. Beneke, G. Buchalla, C. Greub, A. Lenz, U. Nierste, Phys. Lett. B **459**, 631 (1999)
8. S. Narison, A.A. Pivovarov, Phys. Lett. B **327**, 341 (1994)
9. A. Pich, Phys. Lett. B **206**, 322 (1988)

10. For a review, see M. Neubert, Phys. Rep. **245**, 259 (1994)
11. M. Neubert, Phys. Rev. D **45**, 2451 (1992)
12. B. Blok, M. Shifman, Phys. Rev. D **47**, 2949 (1993)
13. T. Huang, C.-W. Luo, Phys. Rev. D **53**, 5042 (1996)
14. C.-S. Huang, Chun Liu, Shi-Lin Zhu, Phys. Rev. D **61**, 054004 (2000)
15. M.S. Baek, J. Lee, Chun Liu, H.S. Song, Phys. Rev. D **57**, 4091 (1998)
16. N. Cabibbo, L. Maiani, Phys. Lett. B **79**, 109 (1978); Y. Nir, Phys. Lett. B **221**, 184 (1989)
17. D. Becirevic et al., Eur. Phys. J. C **8**, 157 (2000)
18. O. Schneider, Talk given at 35th Rencontres de Moriond, Les Arcs, France, 11–18 March 2000, Phys. Rev. D **62**, 083003 (2000)
19. S. Hashimoto, review talk given at the 17th International Symposium on Lattice Field Theory Lattice '99, Pisa, Italy, 29 June–3 July 1999, hep-lat/9909136
20. D. Becirevic, D. Meloni, A. Retico, V. Gimenez, L. Giusti, V. Lubicz, G. Martinelli, hep-lat/0002025
21. S. Hashimoto et al., Phys. Rev. D **62**, 114502 (2000)